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The Effect of Thermal Radiation on MHD Free Convection Boundary Layer Flow over a Plate with Suction and Blowing

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Abstract
In this paper we study the effect of thermal radiation on free convection boundary layer flow over a plate with an applied magnetic field. The suction and blowing is incorporated in the analysis. The system of nonlinear, coupled dimensional partial differential equations governing the flow have been reduced to nondimensional equations by applying suitable similarity transformation and those equations are solved numerically using an implicit finite difference scheme along with quasilinearization technique. From the numerical results we observe that the heat transfer coefficient increases with the increase of radiation parameter both in case of suction as well as injection. Also, In the case of suction, the thermal boundary layer thickness increases while the effect of blowing (injection) is just opposite.

Index Terms: Skin Friction, Heat Transfer, Temperature, Thermal Radiation, Suction, Injection, MHD

1. INTRODUCTION

Convection boundary layer flows are often controlled by injecting or withdrawing fluid through a porous bounding heated surface. This can lead to enhancement heating or cooling of the system and can help to delay the transition from laminar to turbulent. The case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [1]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [2], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection). The effect of suction and injection of freee convection on horizontal plate is investigated by Lin and Yu [3].The effect of strong suction and blowing from general body shapes which admit a similarity solution has been given by Merkin [4]. A transformation of the equations for general blowing (injection) and wall temperature variations has been given by Vedhanayagam et. al. [5].

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Magneto – hydrodynamics (MHD) is the branch of continuum mechanics which deals with electrically conducting fluids and electromagnetic forces. In recent years, MHD flow problems have become more important industrially. Jayakumar.et.al. [6] studied the effect of magnetic field on natural convection boundary layer flow over a plate with suction and blowing. The influence of radiation effect on natural convection flow is important in space and process involving high temperature. Recently, Thermal radiation effect on the axisymmetric boundary layer flow with non uniform slot suction/injection bas been discussed by Poornima [7]. In the present investigation we study the effect of thermal radiation on MHD free convection boundary layer flow over a plate with suction and blowing.

2. MATHEMATICAL FORMULATION

Consider a semi-infinite porous plate at a uniform temperature T_{w0} which is played vertical in a quiescentfluid of infinite extent maintained at constant temperature T_∞. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge in the stream wise direction and y is measured normal to the surface of the plate. The co-ordinate system and flow configuration are shown in Fig.1.

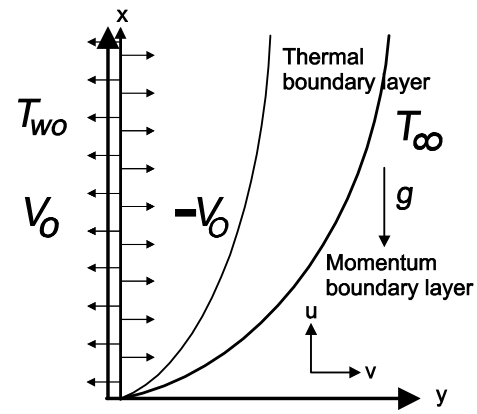

Fig.1. The coordinate system and the physical model.

A magnetic field B_0 is applied in y-direction normal to the body surface and it is assumed that magnetic Reynolds number is small. The Hall current and displacement current effects have been neglected. The fluid is assumed to have constant physical properties. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow.

Under the aforesaid assumptions with Boussinesq's approximation, the equations governing the steady laminar two-dimensional boundary-layer flow are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

The initial and boundary conditions are

$$\begin{aligned}&x=0,\ \ y>0,\quad u=0,\ T=T_\infty\\&y=0;\ \ u=0,\ v=-v_0\,(\mathit{for\ suction}),\\&v=+v_0\,(\mathit{for\,blowing}),\ \ T=T_{wo}\\&y\to\infty\,;\ u=0,\ T=T_\infty\end{aligned} \qquad (4)$$

Here, the radiative heat flux q_r under Roseland approximation, has the form

$$q_r=-\frac{4\sigma^*}{3k}\frac{\partial T^4}{\partial z} \qquad (5)$$

Expanding T^4 in a Taylor series about T_∞ and neglecting higher orders yields:

$$T^4=4T_\infty^3 T-3T_\infty^4 \qquad (6)$$

Substituting (5) and (6) into (3) gives,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(1+N_R)\frac{\partial^2 T}{\partial y^2} \qquad (7)$$

Introducing the following transformations

$$\psi=\frac{\nu^2 g\beta(T_{w0}-T_\infty)\xi^3}{V_0^{\,3}}\left[f(\eta,\xi)\pm\frac{\xi}{4}\right]$$

$$T=T_\infty+\left(T_{wo}-T_\infty\right)G(\eta,\xi)$$

$$\eta=\frac{V_o y}{\nu\xi};\qquad \xi=V_o\left[\frac{4x}{\nu^2 g\beta\left(T_{wo}-T_\infty\right)}\right]^{1/4}$$

$$u=\frac{\partial\psi}{\partial y};\ \ v=-\frac{\partial\psi}{\partial x} \qquad (8)$$

to Eqns.(1) – (3), we see that the continuity Eq.(1) is identically satisfied and Eqns.(2) – (3) reduces, respectively, to

$$F''+G+3fF'-2F^2\pm\xi F'-M\xi^2 F=\xi\left(F\,F_\xi-F'\,f_\xi\right) \qquad (9)$$

$$\Pr^{-1}\left(1+N_R\right)G''+3fG'\pm\xi G'=\xi\left(FG_\xi-G'\,f_\xi\right) \qquad (10)$$

Where

$$u=\frac{V_0^{\,2}4x}{\nu\xi^2}F;\quad v=-\frac{V_0}{\xi}\left(3f+\xi f_\xi-\eta F\pm\xi\right)$$

$$\int_0^\eta F\,d\eta\ \ ;\Pr=\frac{\nu}{\alpha};\qquad M=\frac{\sigma B_0^2}{\rho V_0}$$

$$N_R=\frac{8\sigma T_\infty^3}{3k^2} \qquad (11)$$

It is remarked here that the upper sign in Eqns.(9) and (10) is taken throughout for suction and the lower sign for blowing (injection).

The transformed boundary conditions are

F = 0; G = 1 at $\eta = 0$
F = 0; G = 0 as $\eta \to \infty$
for $\xi \geq 0$ (12)

The local skin friction parameter and heat transfer parameter can be expressed as

$$\tau_w = \frac{V_0}{g\beta(T_{w0} - T_\infty)}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi\left(F'\right)_{\eta=0} \qquad (13)$$

$$Q = \frac{\nu}{V_0\left(T_{w0} - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{1}{\xi}\left(G'\right)_{\eta=0} \qquad (14)$$

Here, u and v are velocity components in x and y direction; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ,η are transformed co-ordinates; ψ and f are the dimension and dimensionless stream functions respectively; Pr is the Prandtl number; ν, α are respectively kinetic viscosity and thermal diffusivity; w_0 and ∞ denote conditions at the edge of the boundary layer and in the free stream respectively and prime $(')$ denotes derivatives with respect to η.

It is worth mentioning here that, when M = 0.0 and N_R = 0.0 the partial differential Eqns. (9) and (10) becomes

$$F'' + G + 3fF' - 2F^2 \pm \xi F' = \xi\left(F\,F_\xi - F'\,f_\xi\right) \qquad (15)$$

$$\Pr^{-1} G'' + 3fG' \pm \xi G' = \xi\left(FG_\xi - G'\,f_\xi\right) \qquad (16)$$

which are exactly same as those of Merkin [2].
Also, It is worth mentioning here that, when $M \neq 0$ and $N_R = 0$ the differential Eqns. (9) and (10) becomes

$$F'' + G + 3fF' - 2F^2 \pm \xi\,F' - M\xi^2 F = \xi\left(F\,F_\xi - F'\,f_\xi\right) \qquad (17)$$

$$\Pr^{-1} G'' + 3fG' \pm \xi G' = \xi\left(FG_\xi - G'\,f_\xi\right) \qquad (18)$$

which are exactly same as those of Jayakumar. et.al [6].

3. RESULTS AND DISCUSSION

The set of partial differential Eqns.(9) and (10) along with the boundary conditions (12) has been solved numerically for $\Pr = 0.7$, by employing an implicit finite difference scheme along with a quasilinearization technique [8], for the sake of brevity, its description is omitted here. In order to assess the accuracy of the method which we have used, results were obtained for M=0.0 by solving Eqns. (15) and (16). The skin friction and heat transfer parameters $\left(\tau_w, Q\right)$ for suction [See Fig.2 (a)] and injection [See Fig.2 (b)] have been obtained and compared with those of Merkin [2]. Further, the steady state skin friction results with magnetic field $M \neq 0$ are compared with those of Jayakumar et al.[6] [See Fig.3(a) for suction] and [See Fig.3(b) for injection] by solving the Eqns.(17) and (18). Our results are found to be in excellent agreement, with the above studies.

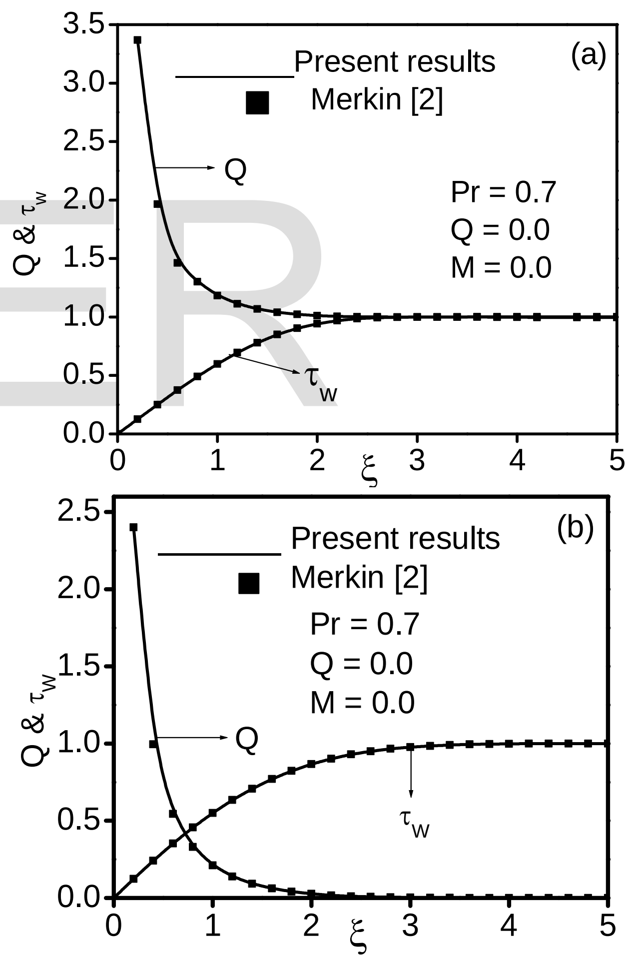

Fig.2. Comparison of steady state skin friction and heat transfer parameters with Merkin [2] for (a) Suction (b) Injection

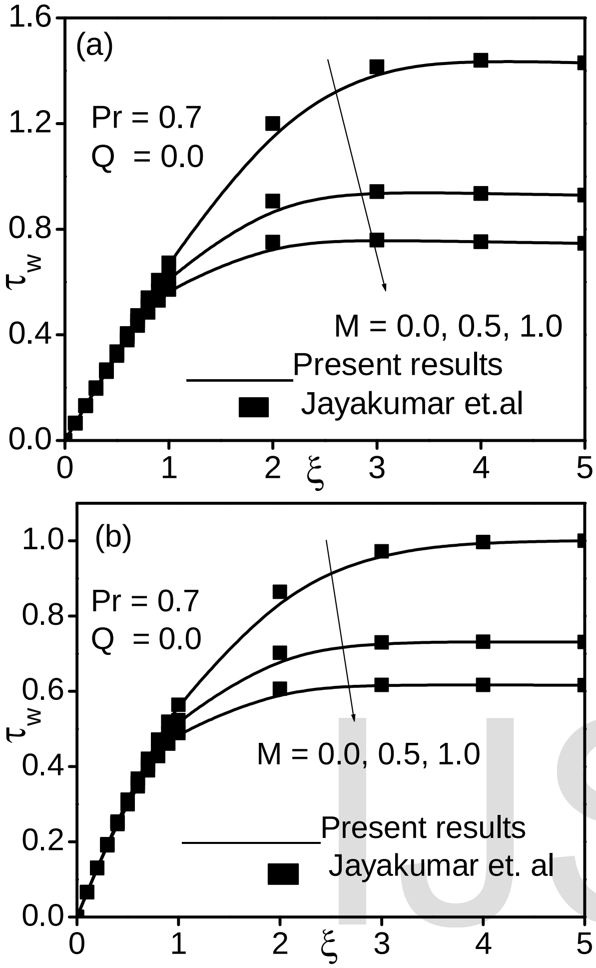

Fig.3. Comparison of skin friction parameter for (a) Suction (b) Injection with those of Jayakumar et al. [6]

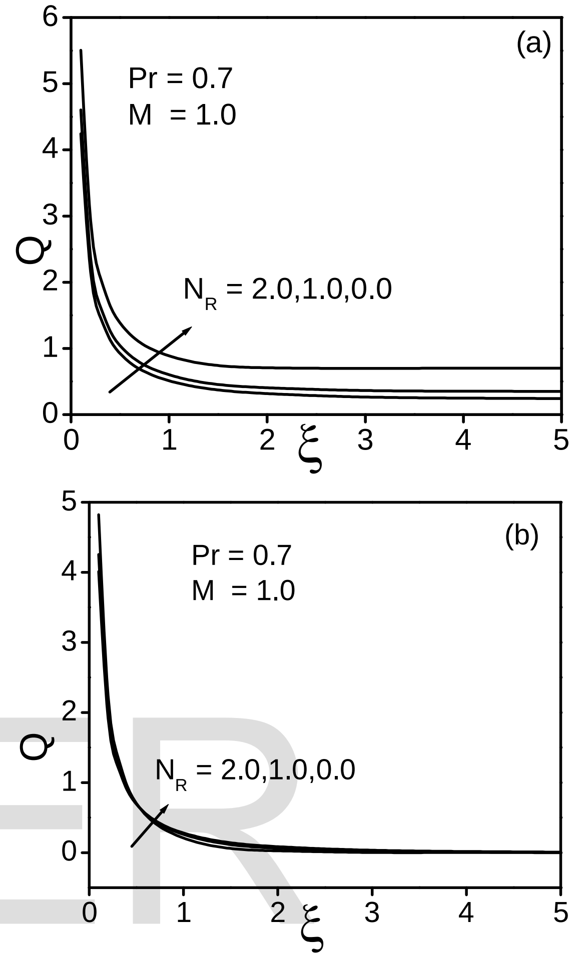

Fig.4. The effect of radiation parameter on heat transfer for (a) Suction (b) Blowing

The effect of thermal radiation *(N_R)* on heat transfer parameter *(Q)* for suction and injection in presence of magnetic field *(M = 1.0)* for air *(Pr = 0.7)* is presented in Fig.4. It is found that, the *Q* decreases with the increase of thermal radiation parameter in case of suction as well as injection. In fact, the percentage of decrease of heat transfer is 37.47% for suction and is 7.13% for injection at $\xi = 1.0$ in the range of N_R ($0.0 \le N_R \le 2.0$)

Fig.5. depicts the effect of thermal radiation *(N_R)* on temperature *(G)* for suction and blowing in the presence of magnetic field *(M = 1.0)* at the stream wise location $\xi = 2.0$. It is observed from the numerical results that the thickness of thermal boundary increases in case of suction while it decreases in case of blowing. Further, the thickness of thermal boundary increases about 25.68% in case of suction while it decreases about 12.09% in case of blowing when computations are done from $N_R = 0.0$ to $N_R = 2.0$ near $\eta = 1.0$.

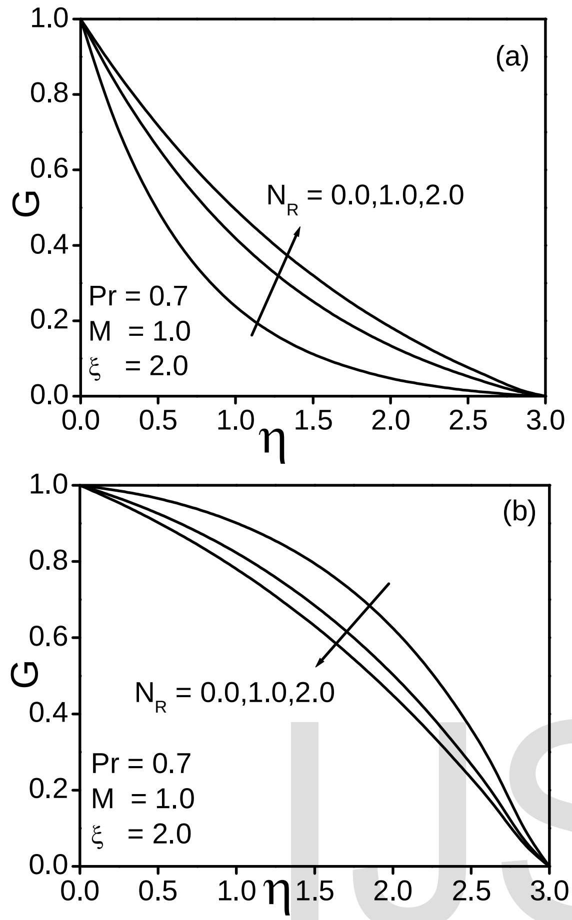

Fig.5. The effect of radiation parameter on Temperature for (a) Suction (b) Blowing

It is remarked here that the skin friction parameter (τ_w) and velocity field (F) is little affected by the thermal radiation parameter (N_R), as it is present only in the energy equation; hence they are not presented here.

4. CONCLUSIONS

In this paper we have studied how the thermal radiation affects the MHD natural convection boundary layer flow from a porous vertical flat plate with suction and injection. The numerical solutions are obtained using a stable, implicit finite-difference method along with a quasilinearization technique. From the present investigation we observe the following:

(i) The influence of thermal radiation decreases the heat transfer in case of suction as well as blowing.

(ii) The thickness of thermal boundary layer increases in case of suction while it decreases in blowing with the increase of thermal radiation parameter.

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